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**Testing Cylindrical Optics****Introduction**

This technical note describes and evaluates strategies for testing cylindrical components, using a Zygo interferometer with standard Zygo interferometer accessories. The analysis and discussion are directed towards measurement of surface irregularity of concave mirrors and wavefront irregularity or positive lenses. Test geometry, data interpretation, alignment and analysis are discussed in each case. Test setups are illustrated, software usage is discussed, and comments are made on the induced aberrations and the interpretation of the interference data. No one technique yields a complete description of the component, yet there is useful information to be gained from each.

Cylinder optics present a more difficult problem for optical testing than do flat or spherical optics. Because cylinders are inherently more difficult to fabricate, it is difficult to manufacture good interferometric references which would allow us to manufacture better cylinders, and so forth. In addition, because cylinders are anamorphic, there is a "clocking" alignment which can increase the complexity of alignment and analysis.

This application note assumes a basic familiarity with interferometry and optical testing. However, some of the concepts are reviewed in the following section.

**Interferometry Fundamentals**

Interferometers operate by generating a reference and test beam of light from a single light source (a laser), causing the test beam to interact with the optic under test, and then re-combining the beams to form an interference pattern (the reference beam reflects from a reference surface). The interference pattern, which consists of light and dark bands called fringes, contains information about the difference between the two beams. Interferometry allows the user to observe very small differences between these beams which may then be related to surface form errors (surfaces) or optical wavefront distortion errors (lenses). Three conditions must be met or understood to let us directly relate the interference pattern to the part under test:

***Common path operation***

The test and reference beams must directly overlay each other inside the interferometer, i.e., they must follow a common path. Since the interference pattern shows us differences, any optical aberrations which are common to both beams are not seen. The only portion of the beam paths which is not in common is the interference cavity. This consists of the surface under test and the reference surface. In the case of testing a lens, an additional surface is also part of the cavity. If the interferometer does not operate in a common path condition, then the interpretation of the interference pattern must take this into account.

### **Reference surface quality**

The reference surface limits the quality of optic which may be tested with the system. The reference surface is part of an accessory which attaches to the interferometer. Standard Zygo reference surfaces are specified as  $\lambda/20$  PV for Transmission Flats (TF), and  $\lambda/10$  PV for Transmission Spheres (TS);  $\lambda = 633$  nm. In some test geometries there are additional surfaces required in the test setup. These other surface will contribute similar errors to the measurement.

### **Interferogram Scale Factor (Wedge Factor)**

The deviation of the interference pattern from the ideal pattern (straight, parallel, equally spaced fringes) is measured in fringes. Converting this to waves of surface error or waves of wavefront error requires using a scale factor which is determined by the geometry of the test setup. Most testing uses a scale factor of 0.5, i.e., 1 fringe =  $\lambda/2$  (1/2 wave). Scale factor is dependent on three parameters of the test setup:

- number of interactions with the part under test
- angle of incidence of test beam on test surface (typically  $0^\circ$ )
- index of refraction in the interference cavity (typically 1)

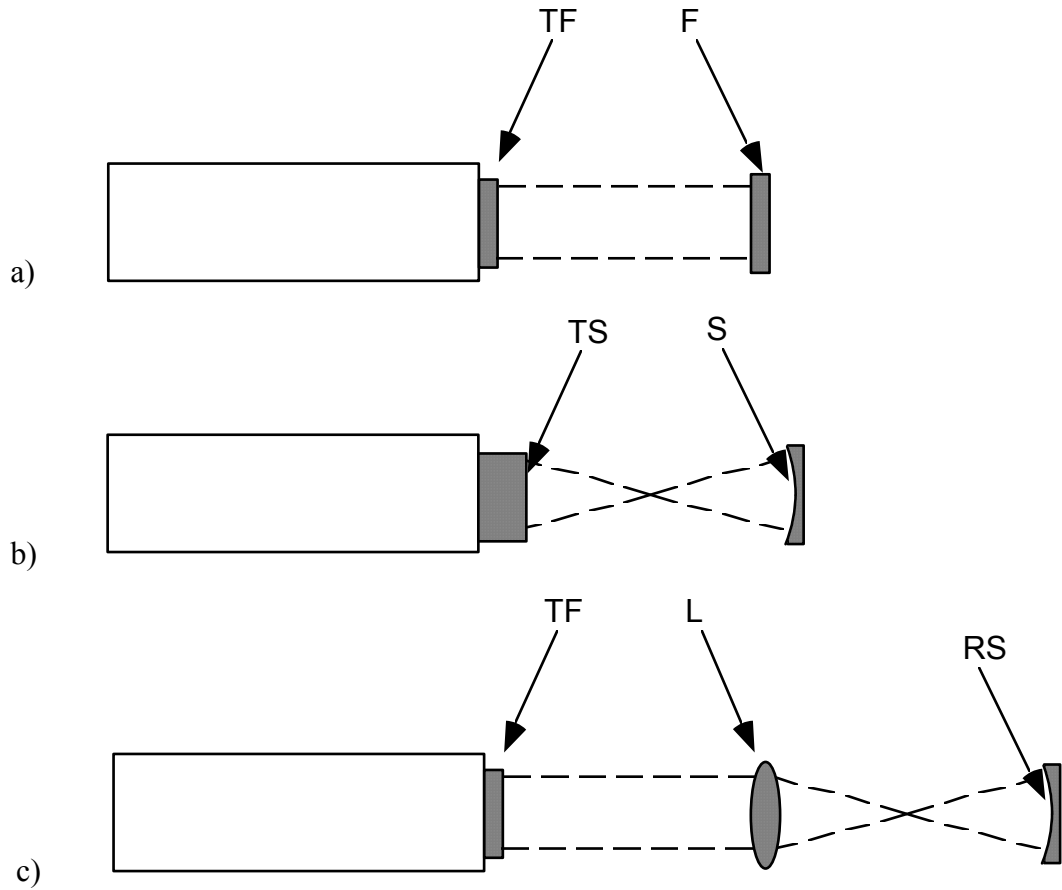
### **Normal Reflection**

In any test setup, the light must reflect back into the interferometer parallel to the path it followed on the way out. If this reflection occurs normal to a surface for all parts of the light beam, then the light follows exactly the same path back into the interferometer as it did on the way out. This is the typical method for testing flat and spherical surfaces and most optical systems. With these test setups, an interference pattern is obtained over the entire test surface/aperture.

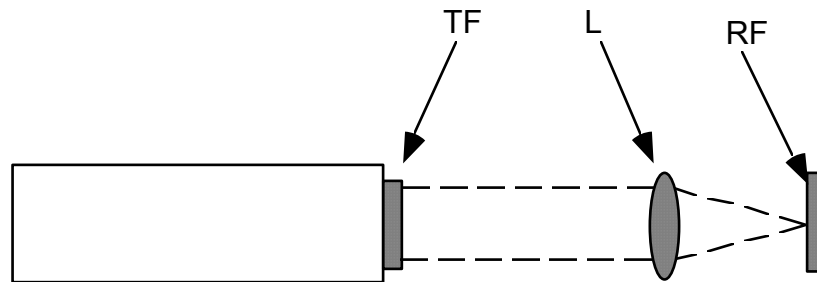
Figure 1 shows test setups for testing of (a) flats surfaces, (b) spherical surfaces, and (c) lenses operating at infinite conjugate. These test geometries all have a scale factor of 0.5 and the interferometer operates in a common path condition. In the case of the lens, an additional surface (RS) is required to reflect the light back. The quality of the RS surface will also limit the accuracy of measurement. The common path operation is insured if proper alignment procedures are used as described in the instrument operating manual.

### **Cat's-eye Reflection**

Some test setups return the light from a cat's-eye reflection. This means bringing the light to focus at a point on the reflecting surface. In this case, the light does not return along the same path. It does return parallel to it, but on the other side of the beam aperture. When this occurs, the interference pattern contains information about the "even" components of the interference cavity and the "odd" components of the internal optics of the interferometer. The other components cancel. "Even" and "odd" refer to the symmetric and anti-symmetric portions relative to the optical axis of the system (see the appendix on odd and even functions). We may still gain information about the component under test, but the interpretation is less straightforward.



**Figure 1.** Test geometries for optical component measurement using a Fizeau interferometer: (a) a flat surface (F) is tested using a Transmission Flat (TF); (b) a spherical surface (S) is tested using a Transmission Sphere (TS); (c) a positive lens (L) is tested at infinite conjugates using a Transmission Flat (TF) and Reference Sphere (RS).



**Figure 2.** Test geometry for testing a positive lens (L) at infinite conjugates using a Fizeau interferometer with a Transmission Flat (TF) and Reference Flat (RF).

## Aberrations - Inherent vs. Fabrication Errors

In some test setups, even if the component is flawless, the interference pattern will show aberrations. This comes from the inherent aberrations of an element used in the optical configuration of that particular test. In this case, it is necessary to see beyond the inherent aberrations to look for the actual fabrication errors. The predominant inherent aberrations which will need to be removed are cylindrical spherical aberration (see appendix) and aberrations which are odd about the cylinder axis. These are discussed later.

## Surface/Lens Specifications

Specifications on optical components can be written from a fabrication standpoint or from a systems standpoint. The optician requires specification which indicate the surface quality of all surfaces and the material quality. The systems engineer requires specifications which allow him to budget the system performance amongst the various components.

One approach to satisfying both needs is to specify components unambiguously on the drawing in terms of systems performance or design tolerancing. The drawing may then refer to an "approved test procedure" (a separate document) which may be used. The vendor should also be given the option to follow another procedure of their choosing, so long as it enables them meet the drawing specifications.

### Surface Specifications

Optical surface specifications are typically given in terms of either:

- "waves" of surface departure from an ideal surface, where the wavelength is specified, or
- "fringes" of deviation as seen with an interference test setup, typically a test plate.

The "fringe" specification can be ambiguous if multiple test setups are being used, some with possibly different scale factors. The "wave" specification is unambiguous since this specifies the surface, not how the surface appears in a particular test. It can be left to the methods group of the optical shop to convert a "wave" specification into a test plate specification.

### Lens Specifications

Lenses are best specified for transmitted wavefront error by the systems engineer. It will be the responsibility of the optical shop or of the designer to budget this error between the individual surfaces of the lens and the material quality.

This wavefront should be specified for the lens as used, typically for "single-pass". That is, the light travels through the lens once in the system. Most interferometric measurements are double-pass. The conversion from a double-pass measurement to single-pass performance is where the scale factor comes in.

## Aperture Specification

Overspecified optics cost more to fabricate. One way in which optics are often overspecified is to apply surface or wavefront quality over a larger area than the beam footprint. For scanner optics, the functional requirements are for minimum surface or wavefront error over a sub-aperture the size of the beam footprint. This is then usually applied to "any sub-aperture within the clear aperture", though some drawings actually specify particular sub-apertures. Most often, the customer and vendor agree on a set of test apertures or include this as part of the "approved test procedure".

## Concave Cylindrical Mirrors

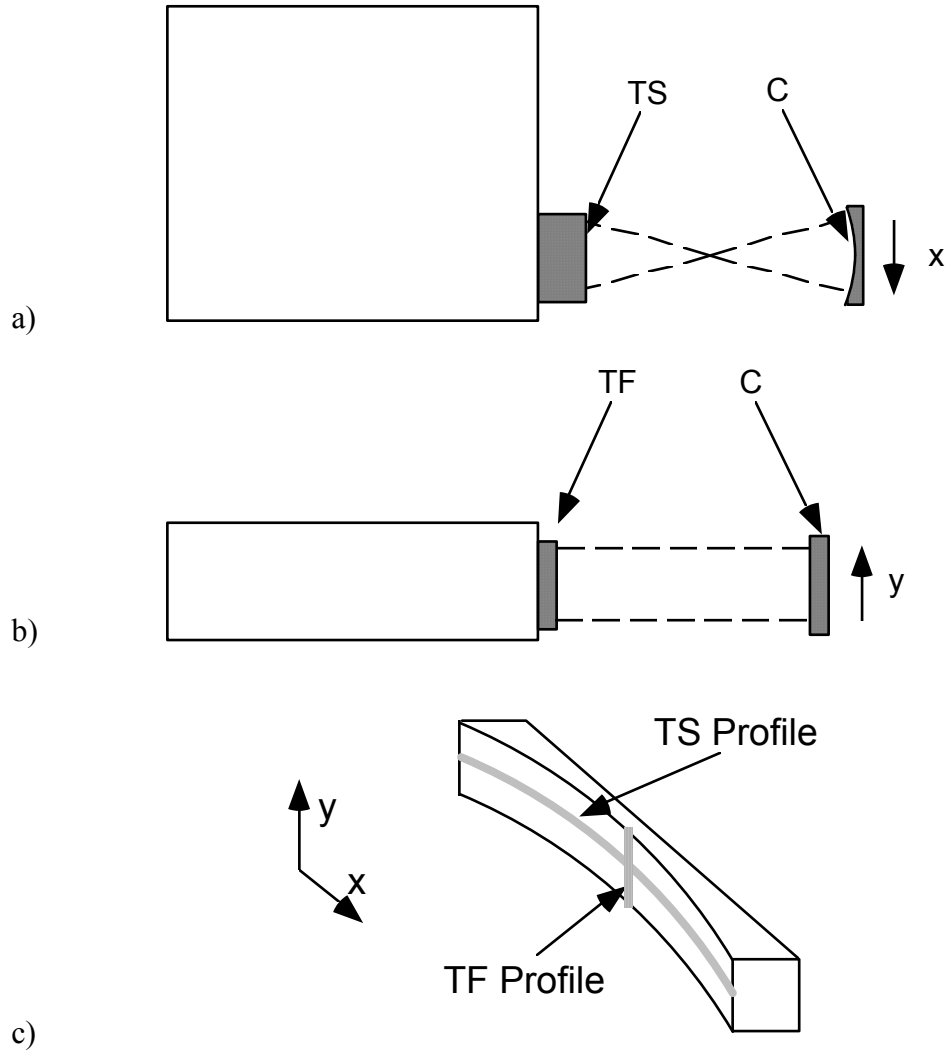
### Single Profile Surface Measurement

#### *Test Geometry*

Using the Zygo interferometer and a Transmission Sphere (TS) will yield interference fringes over a narrow line profile of the surface when tested in a configuration similar to a spherical surface. This profile will be oriented along the surface in the direction with power. Using the interferometer and a Transmission Flat (TF) will yield interference fringes over a narrow line profile of the surface when tested in a configuration similar to a flat surface. This profile will be oriented along the surface in the direction without power. The regions of profiling are indicated in the Figure 3.

#### *Interpretation*

The primary advantage of interpreting profile data measured in this way is that the data is obtained at normal reflection and there are no induced aberrations. The primary disadvantage is that data are obtained only over a profile of the surface. This disadvantage can be minimized by measuring several profiles in each direction over the surface. Surface irregularity and variations in radius of curvature may be evaluated in this way. Twist of the mirror surface cannot be evaluated by measuring profiles.



**Figure 3.** Testing of profiles of a concave cylinder mirror (C) using a Fizeau interferometer: (a) in the focal direction using a Transmission Sphere (TS); (b) in the afocal direction using a Transmission Flat (TF); (c) the profiles indicated on the surface.

***Alignment and positioning - TS***

1. Hold the part in a stress-free mount such that the cylinder axis is aligned with the pixel orientation defined by the interferometer camera.
2. The Transmission Sphere (TS) is aligned to the interferometer using the cross-hair target in the Align mode.
3. Place a flag at the front focus of the TS so that the light passes through the hole in the flag.
4. Place the part at a distance of its nominal radius of curvature from the flag.

5. Observe the reflected light on the flag and position the cylinder so that:
  - the line focus is sharp, and
  - the line focus intersects the hole.
6. Observe the line image in the cross-hair screen and position the cylinder so that:
  - the line focus is sharp, and
  - the line focus intersects the center of the cross-hair.
7. Switch to View mode of the interferometer.
8. Some fringes should be visible along a narrow strip. If so, the flag can be removed. Adjust the tilt and focus (longitudinal position of the part) of the part to minimize the number of fringes. If the fringes have low contrast (highly reflective test part), insert a pellicle to equalize the beam intensities.
9. Zoom in on the fringe pattern
10. Adjust the aperture focus until the region of interference is sharply imaged. If tilt fringes are visible, then the fringes will appear as chevrons when the image is out of focus. As focus is adjusted, the chevrons will straighten and then point the other way. When the chevrons appear straight, the surface is in focus.
11. To determine the location of the profile on the part, slide a flag in front of the part (left to right, or top to bottom) until the fringe pattern is obstructed. The edge of the flag is now at the profiled region.
12. The position of this profile may be moved on the part by re-positioning the part. To reposition a profile seen with a TS (a profile in the direction with power), the part may be either be translated along the cylinder axis or tipped about the center point. This can be done with the interferometer in Align mode so as not to lose alignment completely.

It is desirable to null the fringes prior to measurement.

### ***Degrees of freedom - TS***

The part must be moved in two linear directions: z and the direction perpendicular to the cylinder axis, x. Motion in the y direction is useful to select the position of the profile on the surface. One tilt control is required. This is the tilt about the axis perpendicular to the cylinder axis. The other tilt direction is useful, but not required.

### ***Alignment and positioning - TF***

The alignment procedure is similar to that with the TS except:

- The distance from the part to the interferometer is not critical and no flag is necessary for the alignment.
- If the mirror is highly reflective, then it is recommended that a DynaFlect TF is used. If a Dynaflect TF is not available, then a pellicle may be used.
- The position of this profile may be moved on the part by re-positioning the part. To reposition a profile seen with a TF (a profile in the direction with no power), the part must be rotated about its linear center of curvature. Usually this is accomplished by translation of the part across the beam and then tipping. This can be done with the interferometer in Align mode so as not to lose alignment completely.

**Degrees of freedom - TF**

The part must be moved in two linear directions: x, y. Tilt about an axis perpendicular to the cylinder axis is required. In Figure 3c this corresponds to tilting about the x-axis. The other tilt direction is recommended, but not required.

**Software Analysis**

For the most part, software for this setup is similar to that of simple plano or spherical testing.

**Interferogram Scale Factor**

0.5

**Remove**

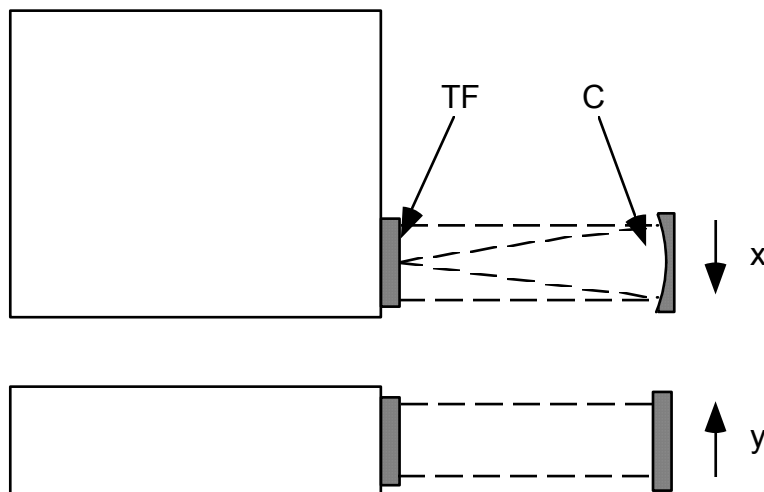
TF - Tilt

TS - Tilt and Focus

**Double-pass Cat's-eye Setup**

**Test geometry**

It is possible to test a concave cylinder as shown in Figure 4. This test setup uses a cat's-eye reflection from the Transmission Flat (TF). The part reflectivity should be high in order to obtain good contrast fringes. Also, a direct reflection from the cylinder mirror must be blocked. This reflection will be very bright and is blocked by placing a narrow opaque strip in the gap between the interferometer mainframe and the TF.



**Figure 4.** Double-pass, cat's-eye test geometry for a concave cylinder mirror (C) using a Fizeau interferometer and Transmission Flat (TF); two views.



### Interpretation

The cat's-eye reflection has the advantage of yielding interference fringes over an area of the test piece. There are three disadvantages:

- The interference pattern contains only the part errors which are *even* about the cylinder axis (where the bright reflection is) and contains interferometer errors which are *odd* about the cylinder axis.
- The cylinder is bringing the light to focus on the TF in a configuration which has inherent aberrations: Cylindrical Spherical Aberration (see Appendix E).
- The measurement area will be limited to the footprint of the 4" beam on the cylinder surface.

The following analysis shows the contributing portions of the interference pattern:

$C(x,y)$  = cylinder surface function, positive surface departure out of the glass; the cylinder axis corresponds to the  $x$  axis.

$TF(x,y)$  = Transmission Flat surface function, positive surface departure out of the glass

$W_i(x,y)$  = Interferometer wavefront error, positive for converging output from interferometer

$W(x,y)$  = measured interference pattern

$$= T(x,y) - R(x,y)$$

$R(x,y)$  = reference beam

$$= 2 \cdot W_i(x,y) + 2n \cdot TF(x,y)$$

$T(x,y)$  = test beam

$$= W_i(x,y) + (n-1) \cdot TF(x,y) + (n-1) \cdot TF(-x,y) + W_i(-x,y) + 2 \cos \theta \cdot [C(x,y) + C(-x,y)] + 2 \cos 2\theta \cdot TF(0,y)$$

$$W(x,y) = +2 \cos \theta \cdot [C(x,y) + C(-x,y)] + 2 \cos 2\theta \cdot TF(0,y) \tag{1}$$

$$+ (n-1) \cdot TF(-x,y) - (n+1) \cdot TF(x,y) + W_i(-x,y) - W_i(x,y)$$

$$= 4 \cos \theta \cdot C_{even}(x,y) + 2 \cos 2\theta \cdot TF(0,y)$$

$$- 2 \cdot TF_{even}(x,y) - 2n \cdot TF_{odd}(x,y) - 2 \cdot W_{i_{odd}}(x,y)$$

For  $C(x,y)$ ,  $TF(x,y)$  and  $W_i(x,y)$  in units of waves,  $W(x,y)$  has units of fringes. The angle  $\theta$  is the angle of incidence of the rays on the cylinder and  $2\theta$  is the angle of incidence on the TF at the cat's-eye point. This varies over the aperture ( $\theta$  is a function of  $x$ ). The *even* and *odd* subscripts refer to the even and odd components of each function with respect to the  $x$  coordinate.

**Effects of the Transmission Flat**

Note the different numeric coefficients in the expression for  $W(x,y)$ . The test is most sensitive to the even component of the cylinder (1/4 wave per fringe). The measurement has sensitivities of 1/2 and 1/3 wave per fringes to the even and odd surface components, respectively (note the factor of  $n$ , index of refraction of the TF). The standard Zygo Transmission Flat is 1/20 wave PV, or about 0.10 - 0.15 fringes in this measurement. A 1/5 wave PV cylinder will have 0.8 fringes error. Transmission Flat can be considered negligible, which yields.

$$W(x,y) \approx 4 \cos\theta \cdot C_{even}(x,y) - 2 \cdot W_{i_{odd}}(x,y) \tag{2}$$

For cylinders better than 1/5 wave PV this effect should be examined more closely.

**Removing interferometer errors**

The error of the interferometer wavefront,  $W_{i_{odd}}$  can either be removed or ignored. The measurement is half as sensitive to this error as to the cylinder surface. The Zygo interferometer wavefront is quality in the range of 1/4 to 1/2 wave PV. This is mostly coma, but the amount of this coma detected in this measurement will depend on the orientation of the interferometer coma relative to the cylinder axis; i.e., it may double or completely cancel. Rotating the clocking of the cylinder it is possible to identify a location which minimizes this error. This is not a practical method since the cylinder axis should line up with the pixel array.

If these errors are to be removed, there are two options:

1. The odd aberrations could be subtracted in polynomial form. If the cylinder axis is along the y axis, then one would subtract the Zernike polynomials which are odd in x: Zernikes 1, 6, 9, 13, 18, 22, 25, 29, 33.
2. The data can be flipped along the cylinder axis and added to itself and then divided by 2.

Subtracting these error leaves us with,

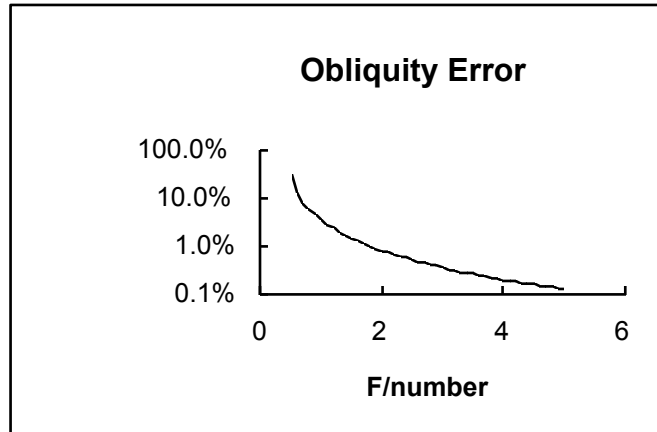
$$W(x,y) \approx 4 \cos\theta \cdot C_{even}(x,y) \tag{3}$$

**Obliquity error**

Obliquity error affects the conversion of fringes to waves, i.e. it is a variation in the scale factor. If there is no error in the part, then this error has no effect. If there is error in the part, or there are fringes due to alignment factors, then this obliquity error will have an effect. Nulling the fringes should minimize this effect. For cylinders slower than  $f/2$  this effect can be ignored (see Figure 5), reducing the measurement to simply,

$$W(x,y) \approx 4 \cdot C_{even}(x,y) \tag{4}$$

Note that  $f/2$  refers to the angle of light being brought to focus as shown in Figure 4, i.e., the focal length divided by the aperture size. Fabricators will use  $f$ /number to designate the radius of curvature divided by the aperture size (also called  $R$ /number). This corresponds to the cone of light in Figure 3a. Since the focal length of the mirror is half the radius of curvature, an  $f/2$  light "cone" in this measurement is produced by what fabricators would call an  $f/4$  surface.



**Figure 5.** Obliquity error due to non-normal incidence of light at the surface. If the obliquity is uniform, it can be corrected by adjusting the scale factor. If the obliquity varies over the aperture, it cannot be corrected.

### **Cylindrical Spherical Aberration**

For a mathematical description of Cylindrical Spherical Aberration, see Appendix E. This aberration is a function of the way in which the mirror is used in this test configuration. It is  $f$ /number dependent, that is, the faster the surface, the greater the aberration.

$$W(x,y) \approx 4 \cdot C_{surface\_even}(x,y) + 2 \cdot C_{csa\_even}(x,y) \tag{5}$$

Fabricating to correct this error will result in an "aspheric cylinder". This error should be subtracted out at this stage. There are two options:

1. Subtract out the nominal Cylindrical Spherical Aberration of the part design from the measurement. This can be accomplished by ray-tracing the test setup with software which will evaluate Zernike polynomials of the wavefront. These polynomial coefficients can then be subtracted from the measurement.
2. Subtract out the "best fit" Cylindrical Spherical Aberration from the measurement. This is accomplished by fitting Zernike polynomials to the data and then subtracting the coefficients as specified in Appendix E.

### **Power**

Cylindrical power perpendicular to the cylinder axis is an artifact of alignment and should be subtracted from the results as should piston and tilt. The cylinder axis location is at the center of the bright reflection which was blocked.

### ***Estimation of odd cylinder components***

We are now left with the even components of the cylinder; even with respect to the cylinder axis position in the image. Cylinder fabrication processes do not tend to produce low order odd surface components. This can lead to the assumption that surface component errors are zero when measuring the entire part. This assumption is not valid when looking at a sub-aperture of the part.

High frequency odd components would be expected to be about the same magnitude at the even components. Also, even and odd components are orthogonal. Two methods of interpretation suggest themselves:

1. For parts measured over a sub-aperture, odd surface components would add in about the same magnitude to the even components. For evaluation of the PV error, the PV surface error would be multiplied by 2 (worst case) or by  $\sqrt{2}$  (root-sum-square). For evaluation of the RMS error, the RMS surface error would be multiplied by  $\sqrt{2}$ .
2. For parts measured over the full aperture, the same analysis could be applied to the higher frequencies. This requires high-pass filtering (convolution filter (sliding window), digital filter (FFT), or fitting to Zernike polynomials ) the data first, then scaling by  $\sqrt{2}$ , and then adding in the low frequency components which were originally filtered out. The resulting surface map is then evaluated for PV and/or RMS.

### ***Alignment and positioning***

1. Hold the part in a stress-free mount such that the cylinder axis is aligned with the pixel array of the interferometer. The rest of this description will assume that the cylinder axis is in the vertical direction (parallel to the y-axis).
2. The Transmission Flat (TF) is aligned to the interferometer using the cross-hair target in the Align mode of the interferometer.
3. Place the cylinder at approximately half its radius of curvature from the TF reference surface. Use a white card to observe the line focus on the TF. Adjust the z position (longitudinal) of the cylinder to bring this in focus.
4. Continue this adjustment using the Align mode of the interferometer and then using the fringes seen in the View mode.
5. Insert the beam stop behind the TF to block the bright reflection.
6. Vertical tilt fringes are removed by adjusting the tilt of the TF. Adjust only the knob which causes the TF to tip about a vertical axis.
7. Horizontal tilt fringes are removed by adjusting the tilt of the part. Adjust only the tilt about a horizontal axis. Continue these three adjustments to minimize the fringe density.
8. If interference fringes are not visible over the entire length of the surface, adjust the part tilt about a vertical axis to maximize the area over which fringes are seen. Some compensation with the other adjustments will be necessary.
9. Place a flag at the part to block part of the beam and adjust the aperture focus until the edge of the flag is sharply imaged.

### ***Degrees of freedom***

The part must be moved in two linear directions: x, z. Motion along the cylinder axis (y) is convenient. Both tilts are required.

### ***Software Analysis***

For the most part, software for this setup is similar to that of simple plano or spherical testing. If subtraction of cylindrical spherical aberration is required, then the software must be capable of fitting Zernike polynomials and generating surfaces based on selected coefficients.

#### **Interferogram Scale Factor**

If the cylinder is being measured for surface error: 0.25.

If the cylinder is being measured for reflected wavefront error: 0.5.

#### **Remove**

TF - Tilt

TS - Tilt and Focus

It is desirable to null the fringes prior to measurement.

## **Convex Cylindrical Mirrors**

This is essentially identical to testing a concave mirror. The primary difference is that the confocal position lies between the TS front focus and the TS itself. This imposes a maximum radius of curvature which can be measured, based on the focal length of the TS.

There are test setups for convex mirrors which use a cat's-eye reflection. These are not discussed in detail here, but are illustrated in the reference by Schnurr and Mann. The discussion of interpretation from the concave cylindrical mirror cat's-eye test also applies to this test configuration for the convex mirror.

Of course, the convex side of a plano convex cylinder lens may be tested using these methods.

## Plano Convex Cylindrical Lenses

Most of this discussion is applicable to all positive power lenses. The restrictions to plano convex relate to some of the alignment issues discussed.

### Single Profile Wavefront Measurement

#### *Test geometry*

Profiles of the transmitted wavefront error of a cylinder lens with positive power can be measured using a test setup as shown in Figure 6. This is similar in many respects to the profiling tests described in the section "Single Profile Surface Measurement" for the concave mirror. The primary limitation is that the afocal profile can be measured only over a size equal to the interferometer beam diameter.

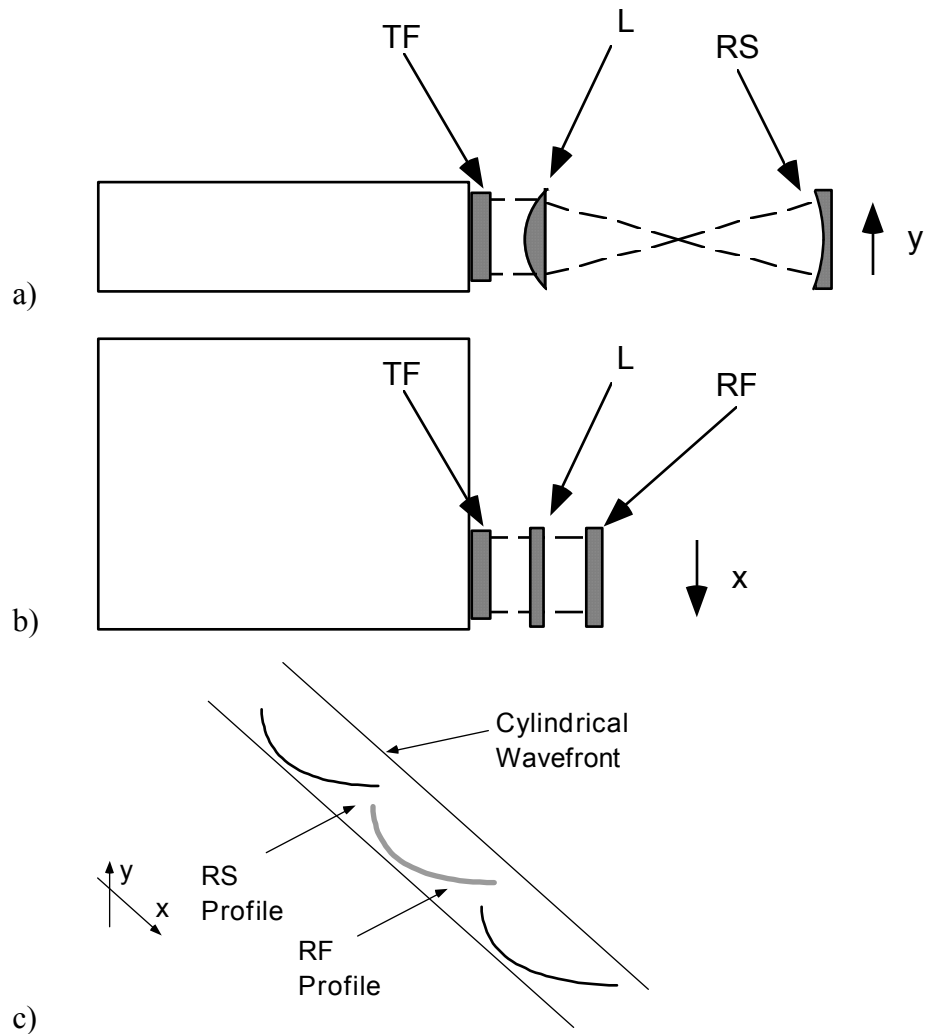
#### *Interpretation*

The primary advantage of interpreting profile data measured in this way is that the data is obtained at normal reflection. The primary disadvantage is that data are obtained only over a profile of the wavefront. This disadvantage can be minimized by measuring several profiles in each direction over the surface. Wavefront irregularity may be evaluated in this way. Twist of the wavefront cannot be evaluated by measuring profiles.

#### *Alignment and positioning*

Alignment of this part is critical. Tilting about the cylinder axis is equivalent to measuring a field point as opposed to on-axis performance. This will contribute the cylindrical version of coma to the wavefront. Tilt in the perpendicular direction has less effect since there is no power in this direction. Figure 6a shows the lens oriented for minimum cylindrical spherical aberration (curved surface facing the collimated light). In order to be certain of the alignment, when the plano side of the lens cannot be referenced to the interferometer, the lens is mounted in a holder which allows the interferometer to reference the plano side (see Figure 7). The alignment tool is fabricated to provide a specular reflection from a surface which is held very parallel to the surface which the plano side of the lens contacts. The degree of parallelism required can be determined from a system level analysis to tilt sensitivity. If this type of tool is used, the lens cannot rest on one of the ground faces, but must fully contact the mounting surface of the alignment tool.

The alignment procedure is similar to the profiling methods found in the section entitled "Single Profile Surface Measurement". Some exceptions are worth noting:



**Figure 6.** Testing of wavefront profiles of a positive cylinder lens (L) using a Fizeau interferometer: (a) in the focal direction using a Transmission Flat (TF) and Reference Sphere (RS); (b) in the afocal direction using a TF and Reference Flat (RF); (c) the profiles indicated on the cylindrical wavefront.

**Lens alignment**

- If there is a reflection from the lens first surface, tilt the lens slightly in the afocal direction (about the y-axis) to reject this reflected light.

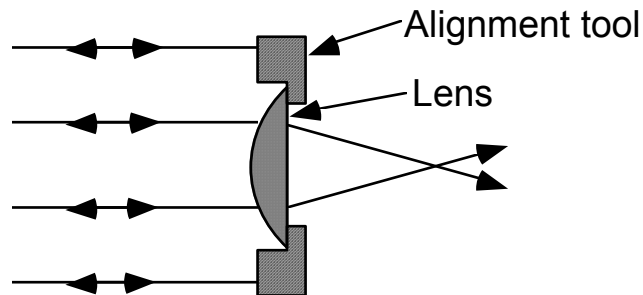
**RS profiling**

- When placing the flag with the hole at the back focus of the lens, there will be a line focus. Place the hole approximately in the middle of this line to start.
- Re-positioning of the profile is best accomplished by translating the lens in the afocal direction (x).

### **RF profiling**

- Keep the distance from the lens to the RF short. If it becomes a sizable fraction of the focal length, there will be ambiguity between the normal and cat's-eye reflections.
- Re-positioning of the profile is best accomplished by tilting the RF about the cylinder axis (x).

It is desirable to null the fringes prior to measurement.



**Figure 7.** Alignment tool to hold plano convex lens in minimum spherical aberration orientation while allowing for alignment relative to the plano side.

### **Degrees of freedom**

- The lens must have both tilt controls as well as the lateral controls: x, y.
- The RF must have both tilt controls.
- The RS must have all three translational adjustments: x, y, z.

### **Software Analysis**

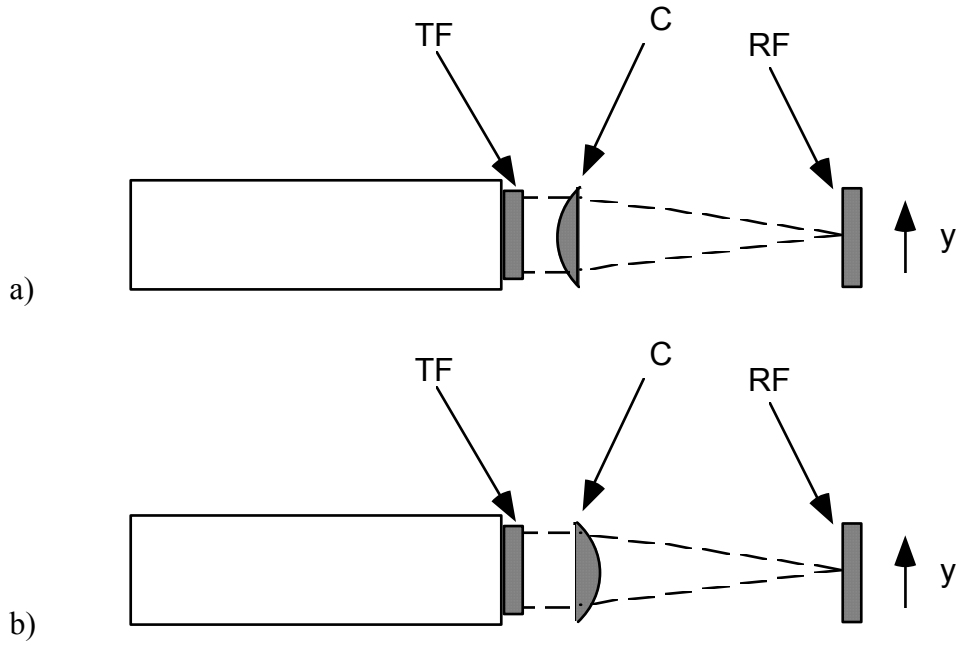
Software for this setup is similar to that of profiling the concave mirror. Simply substitute RS for TS and RF for TF in the various selections of order of fit and aberrations subtracted.

### **Double-pass Cat's-eye Setup**

#### **Test geometry**

The test of a cylinder lens with positive power can be accomplished in transmission making use of a cat's-eye reflection. The test requires an additional flat (RF) which should be a 4% reflective surface (uncoated glass). As shown in figure 6a the plano convex lens is oriented for minimum cylindrical spherical aberration.





**Figure 8.** Double-pass, cat's-eye test geometry for positive cylinder lens (C) using a Fizeau interferometer with a Transmission Flat (TF) and Reference Flat (RF): (a) the plano-convex lens is shown oriented for minimum Cylindrical Spherical Aberration; (b) the plano-convex lens is shown oriented with the flat side towards the TF for alignment ease - this maximizes Cylindrical Spherical Aberration.

### Interpretation

The interpretation of the interference pattern for this test setup is similar to the cat's-eye measurement of the concave mirror if the following additional functions are defined.

$RF(x,y)$  = Reference Flat surface function, positive surface departure out of the glass

$W_L(x,y)$  = Transmitted wavefront error of the lens,

$T(x,y)$  = test beam

$$= W_i(x,y) + (n-1) \cdot TF(x,y) + (n-1) \cdot TF(-x,y) + W_i(-x,y) \\ + W_L(x,y) + W_L(-x,y) + 2 \cos \theta \cdot RF(0,y)$$

$$W(x,y) = W_L(x,y) + W_L(-x,y) + 2 \cos \theta \cdot RF(0,y) \\ + (n-1) \cdot TF(-x,y) - (n+1) \cdot TF(x,y) + W_i(-x,y) - W_i(x,y) \\ = 2 \cdot W_{L_{even}}(x,y) + \cos \theta \cdot TF(0,y) \\ - 2 \cdot TF_{even}(x,y) - 2n \cdot TF_{odd}(x,y) - 2 \cdot W_{i_{odd}}(x,y) \quad (6)$$

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For  $W_L(x,y)$ ,  $TF(x,y)$ ,  $RF(x,y)$  and  $W_i(x,y)$  in units of waves,  $W(x,y)$  has units of fringes. The angle  $\theta$  is the angle of incidence of the rays on the RF at the cat's-eye point.

Once again we can safely ignore the effects of the Transmission Flat. This assumption also holds true for the Reference Flat which is manufactured to similar flatness quality. This lets us write,

$$W(x,y) \approx 2 \cdot W_{L_{even}}(x,y) - 2 \cdot W_{i_{odd}}(x,y) \quad (7)$$

### **Removing interferometer errors**

The interferometer wavefront error,  $W_i$ , are removed in the same way as was described for concave mirrors. The only exception is that for the cylinder axis is along the x axis, then one would subtract the Zernike polynomials which are odd in y: Zernikes 2, 7, 10, 14, 19, 23, 26, 30, 34.

### **Cylindrical Spherical Aberration**

A lens used in this fashion will have some cylindrical spherical aberration

$$W(x,y) \approx 2 \cdot W_{L_{fabrication\_even}}(x,y) + 2 \cdot W_{L_{csa\_even}}(x,y) \quad (8)$$

This aberration may be dealt with in the same manner that was described for the concave mirror and is described in Appendix E.

### **Estimation of odd cylinder components**

As mentioned above, low order odd aberrations are not likely to be created in the cylindrical surface. Low order odd aberrations may genuinely exist in a cylinder lens. These may come from the plano surface or from material inhomogeneity. These aberrations can be kept small by holding a tighter plano surface specification and by using material for which the homogeneity quality is good. If a separate specification is placed on the plano surface, then it is important to remember that the transmitted wavefront is half as sensitive to surface errors (a factor of  $n-1$ ). The inhomogeneity effect can be estimated by  $\Delta n \cdot t$ , where  $\Delta n$  is the material homogeneity specification and  $t$  is the lens thickness.

In order to estimate the odd components of the wavefront, the second strategy which was described for the cylindrical mirror is valid here since the entire width of the lens, in the focal direction, is measured.

### **Power**

Cylindrical power perpendicular to the cylinder axis is an artifact of alignment and should be subtracted from the results as should piston and tilt. The cylinder axis location ( $x=0$ ) can be determined by blocking part of the test beam, until only a narrow strip is left in the interference pattern. This strip lies on the cylinder axis.

***Alignment and positioning***

Alignment of this part is critical as was described for the profiling tests. However, to first order the cylindrical version of coma will be canceled as an odd aberration by the cat's-eye reflection.

1. Hold the part in a stress-free mount such that the cylinder axis is aligned with the detector array of the camera in the interferometer. The rest of this description will assume that the cylinder axis is in the horizontal direction (parallel to the x-axis).
2. The Transmission Flat (TF) is aligned to the interferometer using the cross-hair target in the Align mode of the interferometer.
3. Place the lens in front of the TF and place the RF at the line focus.
4. Remove the lens and align the RF to the TF using the Align mode.
5. Re-position the lens in the beam to bring the line focus on the RF.
6. If the plano side of the lens is facing the TF, align this side to see fringes from the reflection.

If the plano side of the lens is facing the RF, align the reflective face of the alignment tool to see fringes.

7. Minimize tilt and cylindrical power. These are adjusted as follows:  
 Vertical tilt fringes are removed by adjusting the tilt of the RF. Adjust only the knob which causes the TF to tip about a vertical axis.  
 Horizontal tilt fringes are removed by adjusting the tilt of the TF. Adjust only the tilt about a horizontal axis.  
 Cylindrical power is minimized by longitudinal (z) motion of the RF.  
 Continue these three adjustments to minimize the fringe density.
8. If interference fringes are not visible over the entire length of the surface, adjust the part tilt about a vertical axis to maximize the area over which fringes are seen. Some compensation with the other adjustments will be necessary.
9. If the plano side of the lens is facing the TF, adjust the tilt perpendicular to the cylinder axis to so as to eliminate the fringe pattern from the plano surface reflection.
10. Place a flag at the part to block part of the beam at the lens and adjust the aperture focus until the edge of the flag is sharply imaged.

***Degrees of freedom***

The part must have both tilt controls as well as the lateral controls: x, y.

The RF must have both tilt controls as well as the longitudinal control: z.

### Software Analysis

Software for this setup is similar to that of simple plano or spherical testing.

#### Interferogram Scale Factor

0.5

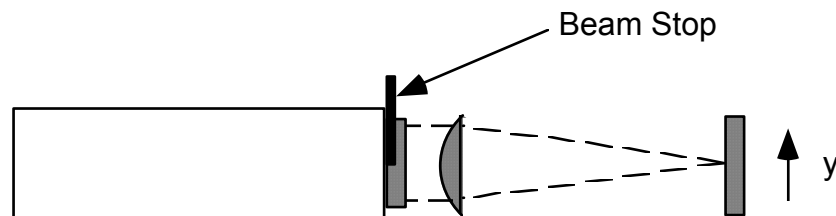
#### Remove

Order of Fit            Plane  
Aberrations Subtracted    Tilt

It is desirable to null the fringes prior to measurement.

### Four Pass Variation

A change in the test geometry removes the interferometer errors, but overlays the two sides of the lens aperture on each other (see Figure 9).



**Figure 9.** Four-pass, cat's-eye test geometry for positive cylinder lens. The beam stop prevents light from re-entering the interferometer. Light is reflected from the TF and re-traces its path.

The interference is visible over half the lens aperture. The measured wavefront still includes only the even components of the lens, but the odd components from the interferometer are no longer measured. Processing requires duplicating the other side of the aperture.

### Recommendation

Again the profiling measurement are recommended. The primary advantage is avoiding the odd/even problems. Specifying and testing surfaces and material is a valid adjunct to the lens test recommended here.

The measurement is limited in its area by the footprint of the interferometer beam on the lens. This can be a drawback if power in the afocal direction needs to be controlled. This footprint can be increased by using aperture converters to allow measurement of the full lens.

## Appendix A

### Methods Requiring Additional Accessories

Through the addition of non-standard accessories to the interferometer, it is possible to simplify the analysis of the interference pattern and to gain more information about the part. Some of the measurements are described here briefly. Some others are included in the references.

#### Concave/Convex Mirror - Transmission Cylinder

Using a Transmission Cylinder (TC) a full map of the surface can be obtained without odd/even ambiguities. The drawbacks to this method are:

- Transmission Cylinders are not very good (typically  $\lambda/2$  -  $\lambda/4$  PV), and
- If alignment of the mirror axis to the TC axis is not well controlled (clocking errors), aberrations will be induced.

Analysis of the interference pattern would use all the standard software except that cylindrical power would need to be subtracted. The scale factor should be set to 0.5.

#### Concave/Convex Mirror - Cylinder Grating

The "Gomez Grating" (currently sold by Reynard) can be used in conjunction with Zygo interferometers to produce a cylindrical test wavefront. This may be used to test cylinder mirrors and yield a full surface map. The same clocking errors are a concern as with the Transmission Cylinder. Grating/system quality is specified as  $\lambda/5$  PV by Reynard. Some users claim to fabricate to  $\lambda/10$  PV. Other users have mentioned grating noise in the interference pattern due to the ruling process for making the grating. Zygo does not have direct experience with these gratings.

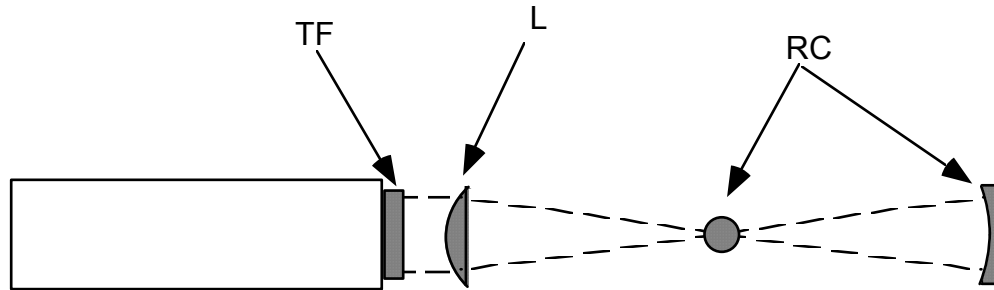
Analysis of the interference pattern would use all the standard software except that cylindrical power would need to be subtracted. The scale factor should be set to 0.5.

#### Plano Convex Lens /Concave Mirror - Fiber Reference

This technique was developed by Joe Geary (currently with Swales & Associates). The fiber assembly is available from a supplier in New Mexico (Geary can supply the contact). Contract measurement is available through Swales. Geary claims that special analysis software is required for this test. Zygo does not have direct experience with this technique. We have a number of reservations about the concept and therefore do not recommend it. See references for details.

### Plano Convex Lens - Reference Cylinder

In the case that a good convex or concave cylinder mirror is available, the lens may be tested as with any other lens in a double-pass transmission setup. There are no odd/even component issues. Only the quality of the Transmission Flat and Reference Cylinder need be considered. A Reference Cylinder can be fabricated to better surface quality than a Transmission Cylinder. Clacking errors would be of concern. The area of the lens measured is limited to the footprint of the interferometer beam on the lens. This may be increased by using an aperture converter.



**Figure 10.** Test geometry for a positive cylinder lens (C) using a Fizeau interferometer with a Transmission Flat (TF) and Reference Cylinder (RC). The reference cylinder may be either concave or convex.

Analysis of the interference pattern would use all the standard software except that cylindrical power and cylindrical spherical aberration would need to be subtracted. The scale factor should be set to 0.5.

## Appendix B

### Comparison of Test Methods

Test	Measurement region	Odd/even complications	Induced CSA	Software Ease <sup>1</sup> of analysis	Ease <sup>1</sup> of alignment	Accuracy	Comments
CC mirror TS	profile	no	none	2	1	$\lambda/10$	multiple profiles recommended aperture limit by TS f/number
CC mirror TF	profile	no	none	2	1	$\lambda/20$	multiple profiles recommended limited to 4" aperture <sup>2</sup>
CC mirror cat's-eye 2-pass	area	yes	yes	4	3	$\lambda/5$ <sup>3</sup>	lose central strip of surface limited to 4" aperture <sup>2</sup>
PCX lens TF/RF	profile	no	no	2	2	$\lambda/10$	limited to 4" aperture <sup>2</sup>
PCX lens TF/RS	profile	no	yes	4	2	$\lambda/5$	limited to 4" aperture <sup>2</sup>
PCX lens cat's-eye 2-pass	area	yes	yes	4	2	$\lambda/5$ <sup>3</sup>	limited to 4" aperture <sup>2</sup>
CC mirror TC	area	no	none	3	3	$\lambda/2$ - $\lambda/4$ <sup>4</sup>	questionable accuracy aperture limited by TC
CC mirror, grating	area	no	none	3	?	$\lambda/5$ <sup>5</sup>	aperture limited by grating, proper selection of grating should allow testing of entire surface
PCX lens RC	area	no	yes	4	3	?	accuracy limited by RC limited to 4" aperture <sup>2</sup>
PCX lens fiber	area	no	yes	4	?	?	limited to 4" aperture <sup>2</sup>
CC mirror fiber	area	no	yes	4	?	?	limited to 4" aperture <sup>2</sup>

<sup>1</sup> Ease is rated on a scale of 1-5; 1 is easy, 5 is hard.

<sup>2</sup> Using aperture converters the size of the measurement area can be increased.

<sup>3</sup> Assumes accurate subtraction of CSA and estimate of odd/even ambiguity.

<sup>4</sup> From industry "common knowledge".

<sup>5</sup> From Reynard specifications.

### ***Software ease of analysis***

The profile techniques are rated 2 because of masking and profile plot alignment. The techniques which require subtracting cylindrical spherical aberration and/or resolving the odd/even ambiguity are rated 4. The techniques which require subtraction of best fit cylindrical power are rated as 3.

### ***Ease of alignment***

None of these tests are particularly hard to align, provided proper tooling for mounting the part has been designed. This is why none rates higher than a 3. The grating and fiber methods are not rated due to Zygo's lack of direct experience with these methods. All methods requiring the clocking of two cylinder axes are rated 3.

### ***Accuracy***

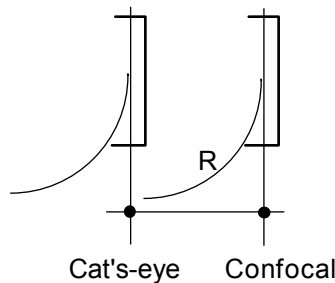
The last three techniques are not rated for accuracy. The accuracy of the Reference Cylinder technique is dependent on reference cylinder surface quality. Swales and Associates claim the fiber techniques are accurate to  $\lambda/15$  PV, but this is the only reference for this technique.



## Appendix C Radius of Curvature Measurement

The cylindrical radius of curvature is evaluated similarly to that of a spherical surface. The part is moved from the cat's-eye position to the confocal, or vice versa, and the distance moved between nulled fringe pattern is measured. The cat's-eye fringe pattern will be a full fringe pattern. The confocal pattern will be along a strip. It is essential that the motion of the cylinder be along the optical axis of the interferometer. This insures that the Cat's-eye point lies on the strip evaluated in the confocal measurement.

Measurement of the radius of curvature at several profiles along the cylinder can indicate variances. Depending upon the technique used, the radius of curvature can be measured to an accuracy of 1µm to 100µm.



**Figure 11.** Radius of curvature measurement geometry.

### Local Radius of Curvature Variation

For a cylindrical mirror, the variation in local radius of curvature can be evaluated from the residual power over a sub-aperture. This is given by,

$$\delta R = \frac{\varphi \lambda}{1 - \sqrt{1 - \left(\frac{d}{2R}\right)^2}} \quad (9)$$

where,

$\delta R$  = variation in cylindrical radius of curvature from the nominal value

$\varphi$  = cylindrical power measured over the sub-aperture in the direction of power (in waves)

$\lambda$  = wavelength of the measurement (633 nm)

$d$  = size of the sub-aperture in the direction of power

$R$  = nominal cylindrical radius of curvature

To make this calculation, the entire surface is measured and the best fit cylindrical power is removed from the entire surface. Then, sub-apertures are evaluated for the residual cylindrical power,  $\varphi$ . The accuracy of determining the size of the sub-aperture will have the greatest effect on the accuracy of this method. This method may be applied to profiles as well. In this case, it is possible to subtract "power".

## Appendix D Twist Measurement

Twist of the mirror surface or of the lens wavefront can be evaluated by comparing tilts of sub-apertures in the direction perpendicular to the cylinder axis. To evaluate this, the entire surface is measured and the best fit cylindrical power is removed. Sub-apertures are then analyzed for tilt.

## Appendix E Cylindrical Spherical Aberration

We will call the cylindrical equivalent of third-order spherical aberration, Cylindrical Spherical Aberration. In the  $x$  direction this is given by  $CSA3_x = C3_x \cdot x^4$ . The cylindrical equivalent of power is  $CPWR_x = C_x \cdot x^2$ . Solving for these in terms of Zernike polynomials, we get

$$CSA3_x = C3_x \cdot x^4 = \frac{C3_x}{8} \left[ Z_{16} + Z_{11} + \frac{1}{2} Z_8 + 3Z_4 + \frac{3}{2} Z_3 + 1 \right] \quad (10)$$

and

$$CPWR_x = C_x \cdot x^2 = \frac{C_x}{2} \left[ Z_4 + \frac{1}{2} Z_3 + \frac{1}{2} \right] \quad (11)$$

The OPD at the edge of the aperture,  $x=\pm 1$ , is matched when  $C_x = -C3_x$ .

The minimum RMS condition for CSA is met when the lower order terms are omitted:

$$CSA3_{x_{min\ rms}} = \frac{C3_x}{8} \left[ Z_{16} + Z_{11} + \frac{1}{2} Z_8 \right] \quad (12)$$

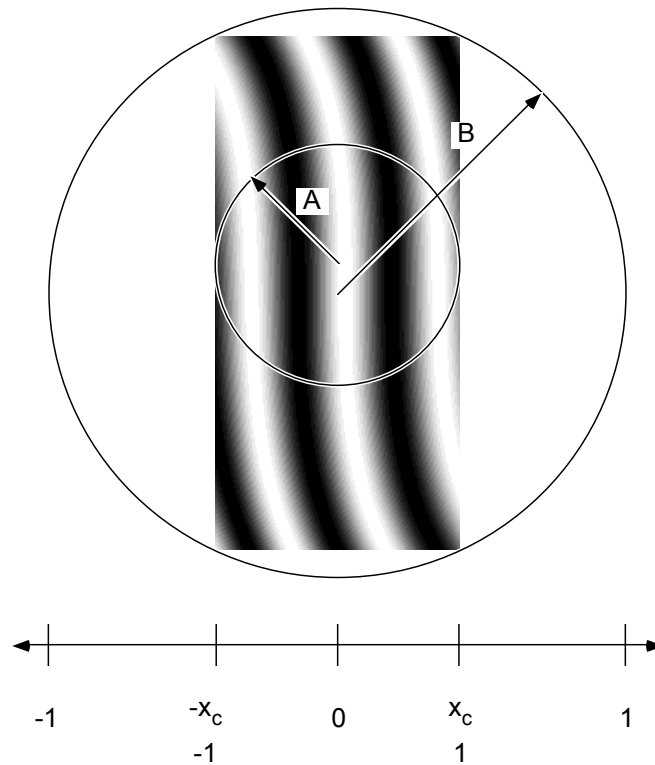
### Aperture considerations

The Zernikes are fit to the circle which encloses the data. It is recommended that this analysis be applied to circular sub-apertures of the part, case A. Thus the position  $x=\pm 1$  will occur within the measured data.

If the circle is to include the entire part, case B, then the coefficients  $C_x$  and  $C3_x$  must be chosen to balance at the edge of the part which corresponds to  $x=\pm x_c = \pm A/B < 1$ . This leads to  $C_x = -C3_x \cdot x_c^2$

### Zernike fit considerations

Fit only that order of Zernikes required to get  $Z_{16}$ , i.e., eighth order. This is not critical for the case of the filled sub-aperture (A), but it is critical for the case of the inclusive aperture (B). There may be other problematic issues with aperture (B) and Zernike polynomials.



**Figure 12.** Defining circular apertures for a rectangular part.

## Subtracting aberrations

### *Subtracting the best fit aberration*

Subtracting the best fit aberration entails subtracting the best fit cylindrical spherical aberration and then subtracting the best fit cylindrical power to yield a minimum RMS wavefront. The sequential operation is intended to minimize any artifacts of using Zernike polynomials on non-circular apertures. This means that if we want to subtract out the best fit cylindrical SA3 from a measurement:

1. Fit Zernike polynomials up to eighth order to the data. This yields coefficients  $A_i$ .

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2. Based on these coefficients, set the following coefficients:

$$A_{16}' = A_{16}$$

$$A_{11}' = A_{16}$$

$$A_8' = \frac{1}{2} A_{16}$$

$$A_4' = 3A_{16}$$

$$A_3' = \frac{3}{2} A_{16}$$

$$A_0' = A_{16}$$

(13)

Set all other coefficients to zero.

3. Subtract these Zernikes from the fit.  
 4. Fit Zernike polynomials up to fourth order (Seidel) to the data.  
 5. Based on these coefficients, set the following coefficients:

$$A_4' = A_4$$

$$A_3' = \frac{1}{2} A_4$$

$$A_0' = \frac{1}{2} A_4$$

(14)

Set all other coefficients to zero.

6. Subtract these Zernikes from the fit.

If the Cylinder is oriented in the other direction, the equations become,

$$CSA3_y = C3_y \cdot y^4 = \frac{C3_y}{16} [2Z_{17} + 2Z_{12} + Z_8 + 6Z_5 + 3Z_3 + 2] \quad (15)$$

and

$$CPWR_y = C_y \cdot y^2 = \frac{C_y}{4} [2Z_5 + Z_3 + 1] \quad (16)$$

The rest of the procedure is similar, except that the coefficient indices are changed.

### **Subtracting a specified amount of SA3**

From a ray-trace of the interferometric test setup, it is possible to evaluate the amount of aberrations expected from measurement. To subtract this aberration, the following procedure is used.

1. Trace the test setup and obtain the wavefront Zernike coefficients.
2. Measure the lens.
3. Subtract a surface defined by these Zernike coefficients from the ray-trace.
4. Fit Zernike polynomials up to fourth order to the remaining data

5. Based on these coefficients, set the following coefficients:

$$A_4' = A_4$$

$$A_3' = \frac{1}{2} A_4$$

$$A_0' = \frac{1}{2} A_4$$

(17)

Set all other coefficients to zero.

6. Subtract these Zernikes from the fit.

A couple of points to be careful of:

- Trace the lens setup in a "single-pass" configuration. Set the scale factor to 0.5.
- Make sure the aperture defined in the ray trace matches the aperture used in fitting the Zernike polynomials to the data.

## Appendix F Odd and Even Surface/Wavefront Components

The *even* and *odd* subscripts refer to the even and odd components of each function with respect to the  $x$  coordinate:

$$C(x,y) = C_{even}(x,y) + C_{odd}(x,y)$$

$$C_{even}(x,y) = C_{even}(-x,y)$$

$$C_{odd}(x,y) = -C_{odd}(-x,y)$$

(18)

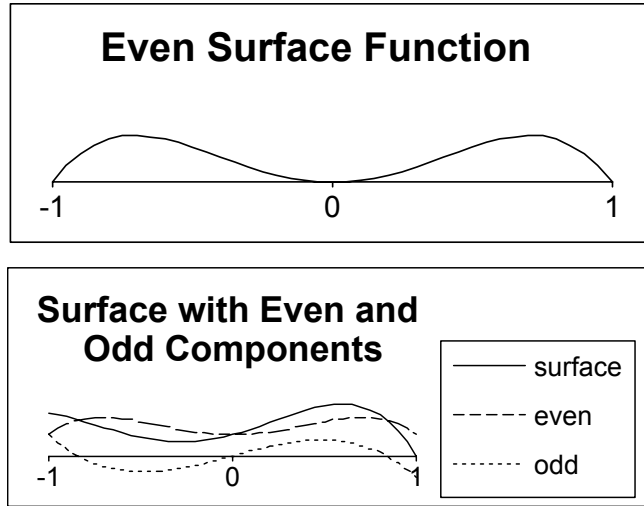
The figures below illustrate the difference between the odd and even surface components.

Even and odd components of a surface may be isolated by the following transformations,

$$C_{even}(x,y) = \frac{C(x,y) + C(-x,y)}{2}$$

$$C_{odd}(x,y) = \frac{C(x,y) - C(-x,y)}{2}$$

(19)



**Figure 13.** Odd and even components of a function: (a) an even function; (b) a function with its odd and even components.

## Appendix G References

There are very few references in the technical literature regarding cylinder testing. Most of them are authored by Joe Geary and the majority of those involve his fiber test.

### Cylinder testing - general

1. Schnurr, A. D. and A. Mann, "Optical figure characterization for cylindrical mirrors and lenses", *Optical Engineering*, Vol. 20, No. 3, pp. 412-416, 1981.

### Cylinder testing - fiber

2. Geary, J. M. and L. J. Parker, "New test for cylindrical optics", *Optical Engineering*, Vol. 26, No. 8, pp. 813-820, 1987.
3. Geary, J. M., "Testing cylindrical lenses", *Optical Engineering*, Vol. 26, No. 12, pp. 1220-1224, 1987.